from different scales of motion or from different constituent components of the source, for example, linear vs nonlinear terms. Such analysis will be presented in future studies.

Acknowledgments

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References

[1]Lighthill, M. J., "On Sound Generated Aerodynamically I. General Theory," *Proceedings of the Royal Society of London, Series A*, Vol. 211, 1952, p. 564.

[2]Colonius, T., Lele, S. K., and Moin, P., "Sound Generation in a Mixing Layer," *Journal of Fluid Mechanics*, Vol. 330, 1997, p. 375.

[3]Mitchell, B., Lele, S., and Moin, P., "Direct Computation of the Sound Generated by Vortex Pairing in an Axisymmetric Jet," *Journal of Fluid Mechanics*, Vol. 383, 1999, p. 113.

[4]Whitmire, J., and Sarkar, S., "Validation of Acoustic Analogy Predictions for Sound Radiated Turbulence," *Physics of Fluids* (to be published).

[5]Crighton, D. G., "Computational Aeroacoustics for Low Mach Number Flows," *Computational Aeroacoustics*, edited by J. C. Hardin and M. Y. Hussaini, Springer–Verlag, New York, 1993, pp. 50–68.

[6]Freund, J. B., Lele, S. K., and Moin, P., "Direct Simulation of a Mach 1.92 Jet and Its Sound Field," AIAA Paper 98-2291, 1998; also *AIAA Journal* (submitted for publication).

[7]Lele, S. K., "Compact Finite Difference Schemes with Spectral-Like Resolution," *Journal of Computational Physics*, Vol. 103, No. 1, 1992, p. 16.

[8]Mohseni, K., and Colonius, T., "Numerical Treatment of Polar Coordinate Singularities," *Journal of Computational Physics* (to be published).

[9]Freund, J. B., "Proposed Inflow/Outflow Boundary Condition for Direct Computation of Aerodynamic Sound," *AIAA Journal*, Vol. 35, No. 4, 1997, pp. 740–742.

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Numerical Optimization of the Suction Distribution for Laminar Flow Control

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Introduction

THE possibility of controlling boundary layers to delay transition is a subject that has received much attention within the aerodynamics research community because of the lower skin-friction drag in laminar flow. It has long been known that small amounts of surface suction can, in theory, greatly enhance the stability characteristics of an attached boundary layer and thereby reduce drag and, hence, operating costs by delaying transition. At Southampton over the past eight years there has been a program of research into the experimental application of distributed suction for the automatic control of boundary-layer transition. In the initial work a plate with two independent suction panels was used, either with or without a freestream pressure gradient. The individual suction flow rates were controlled, maintaining transition at a desired location while minimizing a cost function based on the sum of squares of the suction flow rates,[1,2] which gives a rough approximation to the power consumption of the pumps used in the suction system. This formulation, which is based on the design requirements for a nacelle, gives a nonlinearly constrained optimization problem. To solve it, an algorithm was developed[3] based on a gradient projection method.

In addition to the experimental work, which is detailed in Refs. 1–3, a complementary program of theoretical modeling has been performed. This work was aimed at both modeling the experiments and extending the scope of the research by considering more suction panels and more realistic geometries such as NACA airfoils. The modeling consists of the numerical solution of an interactive boundary-layer formulation for the flow, a linear stability analysis using the Orr–Sommerfeld equation, and transition prediction using the e^N method, followed by an update of the suction flow rates using the same basic strategy as that used in the experiments. As formulated, it mimics the experiments, which enables us to investigate more complex configurations. This Note presents some of the results from the theoretical modeling. Further details of our recent work in this area can be found in Ref. 4.

The calculations described were performed for air at standard conditions ($\nu = 1.5 \times 10^{-5}$ m^{2}s^{-1}) with a freestream velocity of $\tilde{U} = 20$ ms^{-1}, a reference length of $\tilde{L} = 1$ m, and a value of $N_T = 4.3$ in the e^N method, corresponding to a freestream turbulence level of approximately 0.5% as found in the wind tunnel used for the experiments.

Flat Plate Configuration

With no suction, transition is predicted to occur at $x \approx 0.93$, so that any useful suction panel must have at least its leading edge upstream of this position. Calculations were performed using a single suction panel in the region $0.4 \leq x \leq 0.72$. Initially, as suction is applied, the change in x_T is close to linear, with a relatively small amount of suction moving transition a significant distance downstream until $x_t \approx 1.4$, when $C_q = -\tilde{v}_w/\tilde{U} \approx 10^{-4}$. However, as more suction is applied, the slope of the curve changes until the transition asymptotes to a constant position between $x = 1.6$ and 1.65. When this occurs, the boundary layer on the suction panel is extremely thin, and increasing the suction flow rate has little effect on the stability characteristics of the flow downstream of the panel. Hence, for a single panel, after a certain point, further suction effort is largely wasted in terms of delaying the onset of transition. Furthermore, a reasonable estimate of the maximum distance that transition can be moved downstream is obtained by adding the transition position with zero suction to the position of the downstream end of the suction panel ($x_T = 0.93 + 0.72 = 1.65$).

A two-panel configuration, similar to that used in the experiments, was also investigated. The panels were at $0.28 \leq x \leq 0.48$ and $0.58 \leq x \leq 0.78$. Contours of the value of x_T for combinations of the suction flow rates are shown in Fig. 1, as is the contour of the cost function for the optimum value when $x_d = 1.5$. In this case, it is clear that there is a single global optimum for each value of x_d and that for low values of x_d the optimum has essentially the same suction flow rate on both panels. The latter result was also found in the experiments.[2] The transition position and suction flow rates plotted against the iteration number k are shown in Fig. 2. There

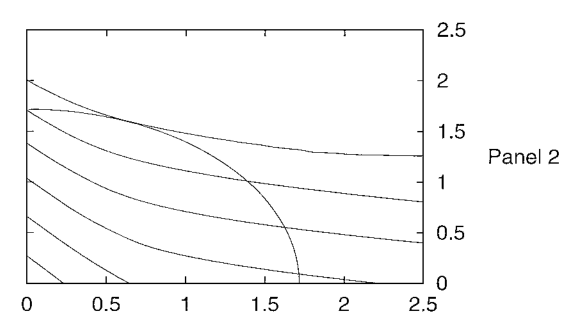

Fig. 1 Transition position for two suction panels as a function of the $C_q \times 10^4$; contours are for constant x_T up to $x_T = 1.5$ in increments of 0.1; also shown is the contour for the optimum value of the cost function when $x_d = 1.5$. The optimum is given by the point closest to the origin where the lines are tangential.

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Table 1 Suction distribution and cost ($C_q \times 10^{-4}$) for the six-panel flat plate

	Panel number						
x_T	1	2	3	4	5	6	Cost
1.7	0.42	0.87	0.73	0.70	0.46	0.00	2.13
2.0	0.31	1.03	0.88	0.76	0.78	0.47	3.33
2.3	0.19	0.90	1.11	1.03	0.92	0.87	4.74
2.6	0.14	0.76	1.47	1.32	1.21	1.11	7.20
2.9	0.26	0.66	1.43	1.61	1.75	1.35	10.04

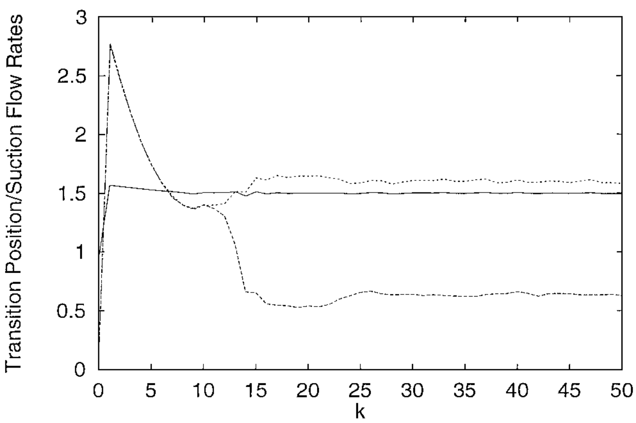

Fig. 2 Transition position x_T and suction flow rates ($C_q \times 10^4$) plotted against iteration number k for two-panel flat plate case: ——, transition position; - - - -, panel one; and · · · ·, panel two.

is an initial overshoot of the transition position, but this is quickly rectified, with the desired position obtained within 10 iterations and the optimum suction values within 15 iterations.

Consider now a plate with six evenly spaced panels, 0.2 long, with a gap of 0.1 between the panels, the leading edge of the first panel at $x = 0.3$, and the trailing edge of the final panel at $x = 2.0$. With this configuration the maximum distance to which the transition position can be forced is $x \approx 3.1$. Suction distributions and cost for six different values of x_d ranging from 1.7 to 2.9 are shown in Table 1. In general, the first panel ($0.3 \leq x \leq 0.5$) makes relatively little contribution to the total suction effort, but as expected the second panel ($0.6 \leq x \leq 0.8$) plays a significant role. The last suction panel ($1.8 \leq x \leq 2$) plays no role when $x_d = 1.7$, as expected, because the transition position is now upstream of the leading edge of this panel. It makes some contribution for $x_d \geq 2$ but never has the largest suction, even with $x_d = 2.9$. As the transition position moves down the plate, the peak suction rate also moves down the plate, from panel 2 when $x_d = 1.7$ to panel 5 when $x_d = 2.9$. As expected the cost increases nonlinearly with the increase in x_d.

Table 1 gives the best solutions obtained over long runs (up to 500 iterations). However, there could be significant variation during the runs and between the runs for different x_T. For $x_d = 1.7$, 2.0, and 2.6, the algorithm converged to a solution in fewer than 30 iterations and held this solution. However, when $x_d = 2.3$, the algorithm is less well behaved, taking more iterations to converge initially and having problems maintaining the solution.[4] This relatively poor behavior arises from the optimization problem not being well defined for $x_d = 2.3$, at least within the numerical tolerance of the algorithm. Even when the algorithm has locked on to the solution, the changes in the individual suction flow rates are at least an order of magnitude greater than those found for $x_d = 2.0$ or 2.6, with variations greater than 20% in the larger flow rates when the predicted value of x_d and the cost are constant to three significant figures. For $x_d = 2.9$, the algorithm was even worse behaved, but this was not surprising because this point is close to the maximum possible transition position and, consequently, in a region where large changes in the suction flow rates are needed to achieve relatively small changes in the transition position.

Plate with a Pressure Gradient

Although the flat plate configuration provides a convenient test problem, for many applications the pressure gradient from the external inviscid flow will be nonzero. Experimentally, a nonzero pressure gradient was generated by placing a constriction in the tunnel on the wall opposite the flat plate. Numerically, this can be mimicked by calculating the inviscid flow past a wall with a hump using potential theory. The hump used is $y(x) = h \cdot f(z)$ with $h = 0.005$ and $f(z) = 1 - 3z^2 + 2|z|^3$ if $|z| \leq 1$ and 0 if $|z| > 1$, where $z = 2(x - 1)$. With this hump there are regions of both favorable and adverse pressure gradient, which, respectively, either locally stabilize or destabilize the flow.

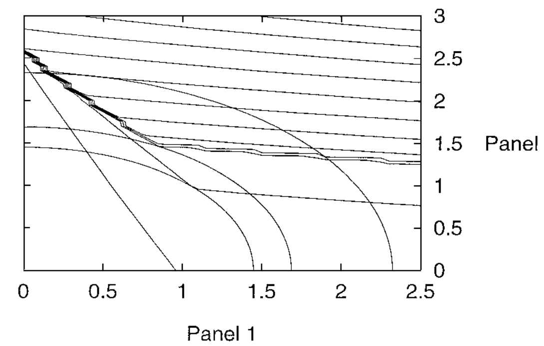

Fig. 3 Transition position for two suction panels as a function of the $C_q \times 10^4$; contours are for constant x_T from 1.3 to 2.4 in increments of 0.1; also shown are a number of contours of constant cost.

A number of runs were performed with this hump and the same six-panel configuration. For this case the natural transition position with zero suction is at $x \approx 1.12$, and the maximum downstream position of transition is $x \approx 3.1$. Not surprisingly, the cost is greater in this case than with the flat plate; for example, for $x_T = 2.6$, $\Sigma\ C_q^2 \approx 1.4 \times 10^{-7}$, approximately double that with no hump.

Although a solution was obtained for $x_d = 2.6$, for many other values of x_d the algorithm completely failed to converge to or hold onto the desired transition position and associated cost once it had found them. Calculations were performed for $x_d = 1.3$–2.7, increasing in steps of 0.1. For $x_d = 1.3$ and 1.4, there was no difficulty in finding a solution; likewise for $x_d = 2.5$ and 2.7. However, for all x_d between 1.5 and 2.4 the algorithm failed to converge.

The reasons for this behavior are best examined using a simpler, two-panel case in which the solution surface can be completely mapped. Consider the case with the panels at $0.56 \leq x \leq 0.92$ and $1.12 \leq x \leq 1.48$. Contours of the transition position vs the suction flow rates plus some of the contours of constant cost are shown in Fig. 3. It can be seen that x_T as a function of the suction flow rates is discontinuous, with the contours of x_T for $x_T \approx 1.5$–2.1 terminating/originating at the discontinuity on the left in Fig. 3. Furthermore, for much although not all of x_T in the range of 1.5–2.0, the minimum cost is at the discontinuity. As can be seen from Fig. 3, for $x_T = 2.0$, although the transition contour terminates at the discontinuity, the minimum cost occurs somewhat to the right of the discontinuity, and numerically the algorithm has no trouble finding the optimum. The optimum solution is easily obtained for $x_d \leq 1.4$ or ≥ 2.0, but for x_d between 1.5 and 1.9 the minimum cost is at the discontinuity, and the algorithm fails in these cases. The discontinuity in the surface arises from a combination of the stabilizing effect of the suction and that of the pressure gradient caused by the presence of the hump, whereby a small increase in suction may move the transition point toward a region of favorable pressure gradient, which when encountered will cause a jump in the transition position.

Conclusions

Provided that the problem is well specified, the optimization method used quickly converges to a solution. By implication, this has been assumed to be the optimum solution, although, except in the two-panel case in which the complete solution surface can be mapped, this has not been formally proved. However, note that, if we start from a different point with nonzero initial suction velocities, the final solution produced is the same as that given. The question of possible local (rather than) global optima should not be ignored in a study of this kind, and it has been found that, in at least one case, the flat plate with six panels and $x_d = 2.3$, there is no single optimum solution within the tolerance of the basic solution algorithm.

Furthermore, for a different problem, with two panels and an imposed pressure gradient, there are regions between the maximum and minimum transition position that are not accessible and other positions for which a solution exists but to which the algorithm will not converge. When the optimum occurs at a discontinuity in the solution surface, the method is bound to fail because the gradient estimation step involves constructing a local linear model of the relationship between the suction flow rates and the transition position. Of course, any solution scheme that relies on local smoothness of the underlying data will also fail in such a condition.

References

[1]Rioual, J.-L., Nelson, P. A., and Fisher, M. J., "Experiments on the Automatic Control of Boundary-Layer Transition," *Journal of Aircraft*, Vol. 31, No. 2, 1994, pp. 1416–1418.

[2]Nelson, P. A., Wright, M. C. M., and Rioual, J.-L., "Automatic Control of Laminar Boundary-Layer Transition," *AIAA Journal*, Vol. 35, No. 1, 1997, pp. 85–90.

[3]Nelson, P. A., and Rioual, J.-L., "An Algorithm for the Automatic Control of Boundary-Layer Flow," Inst. of Sound and Vibration Research, TR 233, Univ. of Southampton, Southampton, England, U.K., 1994.

[4]Tutty, O. R., Hackenberg, P., and Nelson, P. A., "Gradient Projection Methods for Boundary Layer Transition Control," Inst. of Sound and Vibration Research, TR 284, Univ. of Southampton, Southampton, England, U.K., 1999.

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Channel Flow Instability in Presence of Weak Distributed Surface Suction

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Nomenclature

c_i	= amplification rate of Tollmien–Schlichting waves
c_r	= phase speed of flow disturbances
i	= imaginary unit
p_0	= pressure associated with the basic flow
p_1	= pressure associated with flow disturbances
Re	= Reynolds number based on the half-channel height
S_L (S_U)	= amplitude of suction wave at the lower (upper) wall
t	= time
(u, v)	= amplitude function associated with flow disturbances
(u_1, v_1)	= velocity vector associated with flow disturbances
$\|(u, v)\|$	= energy norm associated with flow disturbances; Eq. (6)
$\boldsymbol{V}$	= total velocity vector
$\boldsymbol{V}_0$	= velocity vector associated with the basic flow, (u_0, v_0)
v_L (v_U)	= suction velocity at the lower (upper) wall
x	= streamwise coordinate
y	= normal-to-the-wall coordinate
z	= spanwise coordinate
α	= wave number of wall suction and flow disturbances

I. Introduction

UNIFORM surface suction is an accepted tool for stabilization of laminar boundary layers. In applications the suction distribution is not likely to be constant. Thus, one must consider flow instability in the presence of suction nonuniformities. Whereas spatial distribution of these nonuniformities cannot be predicted apriori, one can assume that their amplitude is of the same order as any other environmental disturbances, as well as of the same order as Tollmien–Schlichting (TS) waves. It is not known how the presence of these nonuniformities may affect flow behavior.

Recently Floryan[1] demonstrated that suction nonuniformities of sufficiently large amplitude induce an instability that manifests itself through generation of streamwise vortices. This instability represents a bypass route to transition that does not rely on the growth of two-dimensional TS waves during its early stages.[2] The formulation given in Ref. 1 assumes suction-induced flow modifications to be larger than flow disturbances, and thus, it cannot resolve issues being addressed in the present study.

The present work is focused on the analysis of the behavior of two-dimensional TS waves in the presence of suction nonuniformities that induce flow modifications of the magnitude comparable to the magnitude of the TS waves. The spatial pattern of the nonuniformities can be either fixed or moving. Because of the linearity of the problem, it is sufficient to consider suction in the form of a single Fourier harmonic and to investigate the effects of variations of its wave number and phase speed. The goals of this work are to determine 1) whether suction nonuniformities affect neutral stability conditions and 2) what type of suction nonuniformities may interfere with the TS waves. The analysis is carried out in the context of plane Poiseuille flow, which represents a standard reference case.

II. Problem Formulation

Consider plane Poiseuille flow confined between flat rigid walls at $y = \pm 1$ and extending to $\pm\infty$ in the x and z directions (Fig. 1). The velocity and pressure fields have the form

$$\boldsymbol{V}_0(x, y) = [u_0(y), 0] = [1 - y^2, 0], \qquad p_0(x, y) = -2x/Re \tag{1}$$

where the fluid motion is directed toward the positive x axis and the Reynolds number Re is based on the half-channel height and the maximum x velocity. At the upper and lower walls, apply suction in the form

$$u_1(x, \pm 1, t) = 0$$

$$v_1(x, -1, t) = v_L(x, t) = \tfrac{1}{2} S_L e^{i\alpha(x - c_r t)} + \mathrm{CC}$$

$$v_1(x, +1, t) = v_U(x, t) = \tfrac{1}{2} S_U e^{i\alpha(x - c_r t)} + \mathrm{CC} \tag{2}$$

where c_r and α denote the phase speed and the wave number of the suction wave, respectively, and CC is the complex conjugate. Because the problem is linear, the response of the flow to an arbitrary suction can be determined by considering only two cases: 1) $S_L = S_U \equiv (S, 0)$, symmetric suction, and 2) $S_U = -S_L \equiv (S, 0)$, asymmetric suction, where S is real. The velocity and pressure fields can be represented as

$$\boldsymbol{V}(x, y, t) = [u_0(y), 0] + [u_1(x, y, t), v_1(x, y, t)]$$

$$p(x, y, t) = p_0(x) + p_1(x, y, t) \tag{3}$$

where (u_1, v_1) and p_1 are the velocity and pressure modifications due to the presence of wall suction. We seek a solution in the

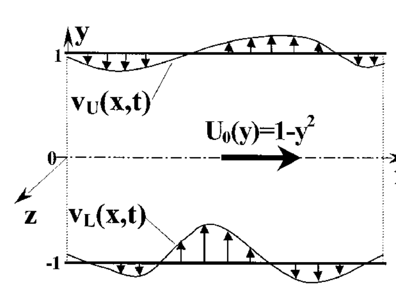

Fig. 1 Flow configuration.

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